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The optimal level of copyright protection

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Abstract

We specify the optimal level of copyright protection for an individual producer and the society as a whole. For an individual producer, the optimal level is (i) no protection, (ii) the level under which the producer's overall profit net of the development cost is zero, or (iii) full protection. The optimal level for the society, on the other hand, critically depends on the distribution of firms' development costs. We also show that an increase in copyright protection may increase or decrease the social welfare loss due to underutilization, while it will always decrease the social welfare loss due to underproduction.

Keywords: Copyright protection; Intellectual property; Underutilization; Underproduction

JEL Classification: D42; K39; L86

1. Introduction

Unauthorized reproduction of intellectual property has long been a concern for producers and policy makers. Producers of intellectual property such as publishers of books and journals, recording companies, and more recently software developers, have consistently claimed that user piracy substantially reduces their revenues, creating a disincentive against the development of better products. For example, a recent study[1] by the software producers claims that the worldwide software piracy

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[1] '2000 Global Software Piracy Report', conducted by International Planning and Research Corporation (IPR) for the Business Software Alliance (BSA) and Software and Information Industry Association (SIIA).

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rate in 1999 is 36%, and the resulting revenue loss is $12 billion.[2] The reason why unauthorized reproduction is claimed to be widespread in such products as books, journals, video and audio tapes, and computer softwares is due to the fact that they have high fixed costs of development and very low marginal costs of reproduction, together with the technological advances that have made it possible for consumers to make high-quality copies of the original works. The recent case of Napster, a small start-up company that offers a technology with which consumers can freely access each other's MP3 music files, is a vivid illustration of this point.

Copyright protection, which gives the original producers various rights including the right to prevent others from making copies, is one of the major institutional measures designed to secure producers' incentive to create useful products in these markets, and thus to 'promote the progress of science and useful arts'.[3] The difficult task of copyright protection, however, is to strike the correct balance between two somewhat contradictory objectives: the first one is to give the right incentive to the producers so that useful works can be created in the first place, and the second one is to promote wide access to and use of the created works so that overall welfare could be improved. To paraphrase the objectives in economists' terms, the first objective is to minimize 'the social welfare loss due to underproduction', and the second objective is to minimize 'the social welfare loss due to underutilization'. The difficulty arises because an increase in copyright protection is generally claimed to (i) decrease the social welfare loss due to underproduction, and at the same time (ii) increase the social welfare loss due to underutilization. In other words, an increase in copyright protection will increase the social welfare by inducing more creative works to be produced, while it will decrease the social welfare by limiting the unauthorized use of the works by consumers.

This paper investigates these claims and derives the optimal level of copyright protection with a simple two-stage model. We first show that, contrary to the common perception, an increase in copyright protection may either increase or decrease the social welfare loss due to underutilization. In particular, we demonstrate that the optimal level of copyright protection for the underutilization problem is either no protection or full protection, depending on the relative magnitudes of producer's marginal cost and the non-substitutability of the copy for the original, which in turn affects the marginal cost of copying. For the underproduction problem, we support the claim that an increase in copyright protection will decrease the social welfare loss due to underproduction. The change, however, is not gradual. We show that the social welfare loss due to

[2] The piracy rate is defined as the ratio of unauthorized copies to the sum of authorized and unauthorized copies.

[3] Article I, Section 8, of the US Constitution. The US Constitution grants the Congress the power 'to promote the progress of science and useful arts, by securing for limited times to authors and inventors the exclusive right to their respective writing and discoveries'.

underproduction is a step function which takes a downward jump to zero at a threshold level of copyright protection.

In addition to investigating the common claims, we specify the optimal level of copyright protection for an individual producer and the society as a whole. For an individual producer, the optimal level of copyright protection is: (i) no protection, (ii) the level under which the producer's overall profit net of the development cost is zero, or (iii) full protection. We provide conditions under which one of these protection levels is an optimal level. We also discuss the optimal level of copyright protection when there exist many firms, each of which may produce a specific copyrightable product. The optimal level of copyright protection for the society crucially depends on the distribution of firms' development costs as well as other features of the environment. See Section 4 for a detailed discussion.

The paper is organized as follows. Section 2 presents related literature. Section 3 sets up the basic model and derives the main results analytically. Section 4 contains discussion and directions for further research. We provide a generalization of the basic model in Appendix A to show the robustness of the main results by numerical methods.

2. Related literature

Given the importance of copyright protection for the performance of the economy, more than ever in the global information economy where worldwide networking is made possible by the explosive growth of the Internet and intangible inputs and outputs, incorporating information and knowledge, constitute the better part of the economic activities, it is surprising that few theoretical frameworks or empirical evidence is accumulated to evaluate the welfare consequences of copyright protection. The existing literature, dating back at least to Plant (1934) and Hurt and Schuchman (1966), did discuss the copyright issues. Most of the papers, however, study the effects of copying per se, rather than the effects of copyright protection, on firm's profit and the social welfare. They are consequently concerned with the clarification of the environments under which firm's profit or the social welfare can be improved even with unauthorized copying. See, for example, Ordover and Willig (1978), Johnson (1985), Liebowitz (1985), Besen (1986), Besen and Kirby (1989), Takeyama (1994, 1997), and Varian (2000).

Earlier works directly concerned with copyright protection or the copyright law are Novos and Waldman (1984) and Landes and Posner (1989). Landes and Posner's work is more of an endeavor designed to deal with a broad set of issues regarding the copyright law, such as the originality requirement for copyright protection, the distinction between ideas and expression, the protection of derivative works, and the issues of fair use. Consequently, it is rather mute on the welfare consequences of increased copyright protection. Novos and Waldman study the welfare effects of increased copyright protection in a model where: (i) the effects of copyright protection are measured by the quality of the product, (ii) consumers have the same valuations for the product, and (iii) consumers' copying costs are higher than the original producer's marginal cost. Their main conclusions are: firstly, the claim that the social welfare loss due to underproduction will decrease with increased copyright protection is partially supported. Secondly, the claim that the social welfare loss due to underutilization will increase with increased copyright protection is not supported.

This paper differs from Novos and Waldman (1984) in several aspects. Firstly, we view the underproduction–underutilization problem of copyright protection in terms of quantity as opposed to quality. In their interpretation, underproduction occurs when the quality of the product deviates from the socially optimal quality. In contrast, the underproduction in our interpretation occurs when a product is not developed (due to insufficient copyright protection) even though it is socially desirable to develop the product. For underutilization, their interpretation of the social welfare loss is the costs incurred when consumers make unauthorized reproduction 'that are in excess of what would be incurred if the good were purchased from the monopolist', while our interpretation is the deadweight loss due to the monopolistic behavior of the producer. In other words, we see that the underutilization occurs when there exist consumers who do not use the product (due to strong copyright protection) but who would have used it under weaker copyright protection. Novos and Waldman's model does not allow consumers to do without the product (even with very strong copyright protection), which is a direct consequence of their assumption that the consumers have the same valuation for the product. Secondly, Novos and Waldman assume that consumers' costs of unauthorized reproduction are always higher than the original producer's marginal cost. We do not impose this restriction. Their restriction has a bite in that their second main conclusion critically depends on this restriction. Lastly, we discuss the optimal level of copyright protection for the society as well as for an individual product, while their discussion is limited to one product.

3. The model

We consider a market for a specific copyrightable product (a software program, for example), which may be produced by a monopolistic producer. The producer incurs a fixed cost of development,[4] denoted by D, if it decides to produce the product; in addition, the producer incurs a marginal cost for each additional unit. The marginal cost is assumed to be constant, and denoted by c. The producer

[4] The fixed cost of development is alternatively called 'the cost of expression' or 'the first-copy cost'. The development cost may be regarded as the expectation of the possible development costs actually incurred.

incurs no cost when it decides not to produce. That is, the producer's total cost is $C(q) = D + cq$ when it produces q (>0) units, and $C(0) = 0$.

There are consumers who are interested in consuming the product. Each consumer consumes either zero unit or one unit of the product. We denote the set of consumers by I, and the valuation (or the maximum willingness to pay) of consumer $i \in I$ by v_i. Consumer $i \in I$ can consume the product by one of the following two methods. The first method is to buy the product from the producer. In this case, the consumer pays the price p, and the net utility is $v_i - p$. The second method is for the consumer to borrow the product from another consumer and make an unauthorized reproduction. We assume that consumers bear no cost when lending the product, but each consumer incurs reproduction costs when making a reproduction.[5] The reproduction costs include the physical costs, like a floppy diskette when making a reproduction of a software program, as well as the inconveniences consumers have to bear in making the copy. We denote consumer i's reproduction costs by z_i. Now, by denoting the level of copyright protection by a parameter y, we assume that each consumer's reproduction costs will increase as the level of copyright protection increases. That is, $\partial z_i / \partial y > 0$ for all $i \in I$.[6] The reason that z_i increases with y is that increased copyright protection increases the likelihood that unauthorized copying will be detected and punished.

The copy is not a perfect substitute for the original. We assume that the gross utility consumer i derives from a copy is proportional to the valuation of the consumer for the original. In other words, we postulate that consumer i's gross utility from consuming a copy is $(1 - \alpha)v_i$, where $0 < \alpha < 1$. The reason a copy is not as good as the original is due to quality degradation, the lack of manuals, the lack of technical supports, and so on. Therefore, the net utility from the consumption of a reproduction is $(1 - \alpha)v_i - z_i$. We let $w_i \equiv \alpha v_i + z_i$. Then, the net utility from the consumption of a reproduction is $v_i - w_i$, and hence w_i may be termed as the *gross reproduction cost*.

We model this production–consumption problem of the copyrightable product as a two-stage game. In the first stage, under the (exogenously given) level of copyright protection, the producer decides whether to develop the product. In the second stage, if the product is developed, consumers choose best choices given their respective reproduction costs and the producer's price, and the producer chooses the optimal price taking account of consumers' choices; otherwise, the game ends with zero payoffs to all parties. Therefore, the first stage is the development stage or the *production* stage, while the second stage is the

[5] Since lenders bear no cost, they do not charge a price. This is so because of the competition between the lenders. Therefore, the model is one of *direct appropriability*, in the terms of Liebowitz (1985) and Besen and Kirby (1989), which does not consider the sharing schemes among consumers. See Section 4 for a detailed discussion.

[6] There are three dimensions in which the level of copyright protection can be increased—length, breadth, and enforcement. While y may represent any of these dimensions, it is probably most appropriate to interpret y as the level of enforcement.

consumption stage or the *utilization* stage. Note that the producer's development cost D is incurred in the first stage, and the marginal cost is incurred in the second stage. In the following, we first analyze the second stage where the pricing and consumption decisions of the already-developed product occur, and then move to the first stage where the development decision occurs.

3.1. The second stage: utilization

Suppose the product is developed in the first stage. Since the development cost is sunk in the second stage, we can ignore this cost when we analyze the underutilization problem of the second stage. We start with the analysis of consumers' behavior. A typical consumer, say consumer $i \in I$, will make the following choices depending on the relative magnitudes of (v_i, w_i, p).

(a) $p = \min\{v_i, w_i, p\}$: purchase from the producer
(b) $w_i = \min\{v_i, w_i, p\}$: unauthorized reproduction
(c) $v_i = \min\{v_i, w_i, p\}$: no consumption.

Given the price p and the level of copyright protection y, we can define the set $B(p; y) = \{i \in I \mid p \le v_i, p \le w_i\}$ of consumers who buy the product from the producer, the set $U(p; y) = \{i \in I \mid w_i \le v_i, w_i < p\}$ of consumers who make reproductions, and the set $N(p; y) = \{i \in I \mid v_i < p, v_i < w_i\}$ of consumers who neither buy nor make reproductions.[7]

Then we have our first result.

Proposition 1. *As the level of copyright protection y increases, given that other things remain the same,* (*i*) $U(p; y)$ *is non-increasing, and* (*ii*) $B(p; y)$ *and* $N(p; y)$ *are non-decreasing. To state more formally, for all* $y' > y$, *we have*

$$U(p; y') \subseteq U(p; y); \quad B(p; y) \subseteq B(p; y'); \quad N(p; y) \subseteq N(p; y').$$

Therefore, the producer's profit is non-decreasing.

Proof. The inclusion relationship of the sets can be deduced from the analysis of the cases (a)–(c) above. The conclusion of non-decreasing profit can be derived from the fact that $B(p; y)$ is non-decreasing. Specifically, if we denote the profit-maximizing level of price by p and the corresponding profit by π when the level of copyright protection is given by y, and the profit-maximizing level of

[7] We assume that consumers choose to {buy, make reproductions, buy, resp.} when they are indifferent between {buying and making reproductions, making reproductions and no consumption, buying and no consumption, resp.}.

price by p' and the corresponding profit by π' when the level of copyright protection is given by y', then, from the fact that $B(p; y) \subseteq B(p; y')$, we have $\pi = (p - c) \cdot |B(p; y)| \leq (p - c) \cdot |B(p; y')| \leq (p' - c) \cdot |B(p'; y')| = \pi'$. In the previous sentence, $|B|$ denotes the measure of consumers in the set B. □

For a concrete analysis, we now assume that consumers' valuations are uniformly distributed over the unit interval [0, 1]. Thus, we can identify the set I of consumers with the set V of valuations, which is the unit interval. (It is straightforward to relax this assumption, and impose an arbitrary distribution function on [0, 1].) In addition, we assume that, for any given level of copyright protection, consumers' reproduction costs are the same. That is, $z_i(y) = z(y)$ for all i and y. (We relax this assumption in Appendix A to show that the main results of the paper are independent of this assumption.) With this assumption, we will identify the level of copyright protection (y) with consumers' reproduction costs (z) for the rest of the paper. Observe that this is only for expositional clarity. Then, consumers' optimal choices given the price and the level of copyright protection can be summarized as follows:

Proposition 2 (Consumer choice).

(i) $p < z/(1-\alpha)$: *in this case, consumers who belong to* $[0, p)$ *do not consume the product, and consumers who belong to* $[p, 1]$ *consume the product by purchasing from the producer. That is,* $B(p; z) = [p, 1]$; $U(p; z) = \emptyset$; *and* $N(p; z) = [0, p)$.

(ii) $p \geq z/(1-\alpha)$: *in this case, consumers who belong to* $[0, z/(1-\alpha))$ *do not consume the product, consumers who belong to* $[z/(1-\alpha), (p-z)/\alpha)$ *consume the product by making reproductions, and consumers who belong to* $[(p-z)/\alpha, 1]$ *consume the product by purchasing from the producer. That is,* $B(p; z) = [(p-z)/\alpha, 1]$; $U(p; z) = [z/(1-\alpha), (p-z)/\alpha)$; *and* $N(p; z) = [0, z/(1-\alpha))$.

Observe that the first case is such that no consumer has an incentive to make a reproduction since the price is set low enough. The proposition can be easily proved using the following diagram. We will explain the second case. The first case can be similarly explained.

As discussed above, consumers make choices depending on the relative magnitudes of the valuation v_i, the gross reproduction cost $w_i = \alpha v_i + z$, and the price p. In Fig. 1, we see that consumers who belong to $[0, z/(1-\alpha))$ have $v - p < 0$ and $v - w < 0$, choosing no consumption. Likewise, consumers who belong to $[z/(1-\alpha), (p-z)/\alpha)$ have $p - w > 0$ and $v - w \geq 0$, choosing to make reproductions, and consumers who belong to $[(p-z)/\alpha, 1]$ have $v - p \geq 0$ and $w - p \geq 0$, choosing to buy from the producer.

The demand function for the copyrightable product derived from the consumers' choices is given as follows:

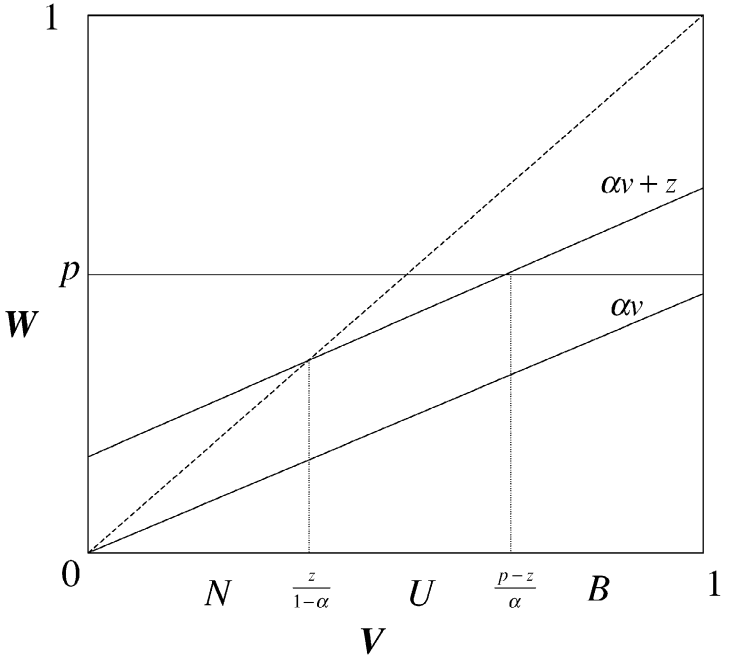

Fig. 1. Consumer choice.

$$D(p) = \begin{cases} 1 - (p - z)/\alpha; & \text{when } p \geq z/(1 - \alpha), \\ 1 - p; & \text{when } p < z/(1 - \alpha). \end{cases}$$

We make the following assumption on the parameters.

Assumption 1. $c \leq \alpha + z \leq 1$.

If $\alpha + z > 1$, then $w \equiv \alpha v + z > v$ holds for all $v \in [0, 1]$. This means that, for every consumer, the gross reproduction cost exceeds the valuation for the product. Thus, there exists no unauthorized reproduction of the copyrightable product, which is not a meaningful situation to consider. If $c > \alpha + z$, then the marginal cost of the producer is higher than any consumer's gross reproduction cost. Thus, no consumer will buy the product from the producer. This again is not a meaningful situation to consider. Therefore, Assumption 1 is a very weak assumption. We will denote the minimum possible level of z as $\underline{z} \equiv \max\{0, c - \alpha\}$.

Under the assumption, the marginal revenue function of the producer is given as

$$MR(q) = \begin{cases} \alpha + z - 2\alpha q; & \text{when } q \leq 1 - z/(1 - \alpha), \\ 1 - 2q; & \text{when } q > 1 - z/(1 - \alpha), \end{cases}$$

and we can derive the profit-maximizing price p, the profit-maximizing quantity q, and the resulting profit π as in the following proposition.

Proposition 3 (The equilibrium outcome). (i) $\underline{z} \le z \le (1-\alpha)(\alpha+c)/(1+\alpha)$: *in this case, the equilibrium outcome is given by*

$$\{p, q, \pi\} = \{(\alpha+z+c)/2, (\alpha+z-c)/2\alpha, (\alpha+z-c)^2/4\alpha\},$$

and consumers' choices are given as follows:

$$B(p;z) = [(\alpha+c-z)/2\alpha, 1]; U(p;z) = [z/(1-\alpha), (\alpha+c-z)/2\alpha);$$
$$\textit{and } N(p;z) = [0, z/(1-\alpha)).$$

(ii) $(1-\alpha)(\alpha+c)/(1+\alpha) < z \le (1-\alpha)(1+c)/2$: *in this case, the equilibrium outcome is given by*

$$\{p, q, \pi\} = \{z/(1-\alpha), 1-z/(1-\alpha), (1-z/(1-\alpha))(z/(1-\alpha)-c)\},$$

and consumers' choices are given as follows:

$$B(p;z) = [z/(1-\alpha), 1]; U(p;z) = \emptyset; \textit{ and } N(p;z) = [0, z/(1-\alpha)).$$

(iii) $(1-\alpha)(1+c)/2 < z \le 1-\alpha$: *in this case, the equilibrium outcome is given by*

$$\{p, q, \pi\} = \{(1+c)/2, (1-c)/2, (1-c)^2/4\},$$

and consumers' choices are given as follows:

$$B(p;z) = [(1+c)/2, 1]; U(p;z) = \emptyset; \textit{ and } N(p;z) = [0, (1+c)/2).$$[8]

The consumer surplus CS and the social welfare SW for the first case in the proposition are given by the following expressions.

$$CS = \int_{z/(1-\alpha)}^{(\alpha+c-z)/2\alpha} (v - \alpha v - z)\,\mathrm{d}v + \int_{(\alpha+c-z)/2\alpha}^{1} (v - (\alpha+z+c)/2)\,\mathrm{d}v$$
$$= \frac{1}{8}\left(4 - 3\alpha - 2c + \frac{c^2}{\alpha}\right) - \frac{1}{4}\left(3 + \frac{c}{\alpha}\right)z + \frac{1+3\alpha}{8\alpha(1-\alpha)}z^2,$$

[8] Since the proof is straightforward, albeit rather lengthy, we will leave it to the reader.

$$SW = CS + \pi = \int_{z/(1-\alpha)}^{(\alpha+c-z)/2\alpha} (v - \alpha v - z)\,\mathrm{d}v + \int_{(\alpha+c-z)/2\alpha}^{1} (v - c)\,\mathrm{d}v$$
$$= \frac{1}{8}\left(4 - \alpha - 6c + \frac{3c^2}{\alpha}\right) - \frac{1}{4}\left(1 + \frac{3c}{\alpha}\right) z + \frac{3+\alpha}{8\alpha(1-\alpha)} z^2.$$

We analyze the effects of the change in the level of copyright protection on the profit, the consumer surplus, and the social welfare below. First of all, we can easily see that $\partial \pi / \partial z > 0$. Therefore, the producer's profit is strictly increasing as the level of copyright protection increases. On the other hand, since

$$\frac{\partial CS}{\partial z} = \frac{3\alpha z + z}{4\alpha(1-\alpha)} - \frac{1}{4}\left(3 + \frac{c}{\alpha}\right) \le \frac{1}{4} \cdot \frac{4\,\alpha z + (1-\alpha)z}{\alpha(1-\alpha)} - \frac{1}{4}\left\{2 + \frac{z(1+\alpha)}{\alpha(1-\alpha)}\right\}$$
$$= \frac{1}{2}\left(\frac{z}{1-\alpha} - 1\right) < 0$$

for $z \le (1-\alpha)(\alpha+c)/(1+\alpha)$, the consumer surplus is strictly decreasing as the level of copyright protection increases. As for the social welfare, we first observe that

$$\frac{\partial SW}{\partial z} = \frac{1}{4} \cdot \frac{\alpha z + 3z}{\alpha(1-\alpha)} - \frac{1}{4}\left(1 + \frac{3c}{\alpha}\right)$$

is a strictly increasing function of z over the interval $[0, (1-\alpha)(\alpha+c)/(1+\alpha)]$. We also find that (i) $\partial SW/\partial z = -1/4 \cdot (1 + 3c/\alpha) < 0$ when $z = 0$, and (ii) $\partial SW/\partial z = 1/2 \cdot (1 - z/(1-\alpha)) > 0$ when $z = (1-\alpha)(\alpha+c)/(1+\alpha)$.

Summarizing the discussion, we have

Proposition 4. *For* $z \in [\underline{z}, (1-\alpha)(\alpha+c)/(1+\alpha)]$,
(i) *the profit* π *is a strictly increasing function of* z;
(ii) *the consumer surplus* CS *is a strictly decreasing function of* z, *and*
(iii) *the social welfare* SW *strictly decreases on the interval* $[0, (1-\alpha)(\alpha+3c)/(3+\alpha))$, *attains the minimum at* $z = (1-\alpha)(\alpha+3c)/(3+\alpha)$, *and strictly increases on the interval* $((1-\alpha)(\alpha+3c)/(3+\alpha), (1-\alpha)(\alpha+c)/(1+\alpha)]$.

The behavior of the social welfare is due to two countervailing effects that an increase in the protection level has on welfare. Firstly, an increase in the protection level increases consumers' reproduction costs. This will decrease the welfare because consumers who make reproductions have to bear a higher cost. Mathematically, this effect is given by $-((\alpha+c-z)/2\alpha - z/(1-\alpha))$. Secondly, an increase in the protection level will induce some consumers to switch from making reproductions to buying from the producer. This will increase the welfare because the consumption is obtained by a more efficient method, namely, by the production of the monopolistic producer. The reason that the production of the monopolistic producer is more efficient is that, for the marginal consumers who switch as a result of a small increase in the protection level, the gross reproduction cost $w = \alpha v + z$ is almost the same as the monopolistic producer's p, which is greater than the marginal cost c. Mathematically, this effect is given by $(\alpha + z - c)/4\alpha$. When z is low, the first effect dominates, resulting in social welfare decrease; when z is high, the second effect dominates, resulting in social welfare increase.

We can easily show that $\partial\pi/\partial z > 0$, $\partial CS/\partial z < 0$, and $\partial SW/\partial z < 0$ for the second case, and $\partial\pi/\partial z = \partial CS/\partial z = \partial SW/\partial z = 0$ for the third case of Proposition 3. Let us define z^M to be the level of copyright protection at which the social welfare attains an interior maximum, that is, $z^M = (1-\alpha)(\alpha + c)/(1+\alpha)$. Note that z^M is the minimum level of copyright protection under which no unauthorized reproduction exists. Please refer to Fig. 2 for the shapes of the profit and the social welfare functions.

Contrary to the common perception that an increase in copyright protection will increase the social welfare loss due to underutilization, the opposite may sometimes hold. From the above discussion, we observe that the optimal level of copyright protection is no protection if $SW(\underline{z}) \geq SW(z^M)$, and full protection if $SW(\underline{z}) < SW(z^M)$. When $\underline{z} > 0$, that is, when $c > \alpha$, it can be easily shown that $SW(\underline{z}) - SW(z^M) = \alpha(1+3\alpha)(1-c)^2/2(1-\alpha)(1+\alpha)^2 \geq 0$. Therefore, the opti-

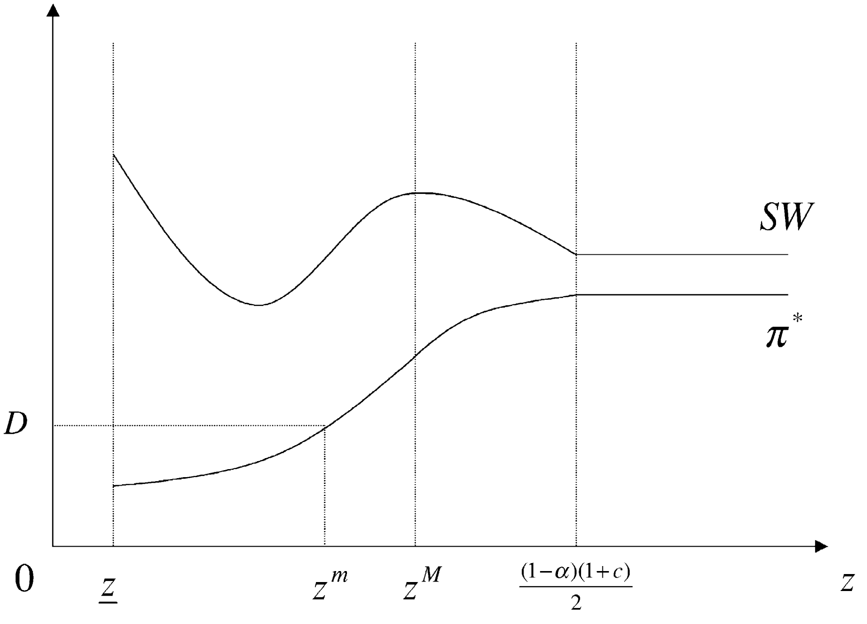

Fig. 2. The profit and the social welfare.

mal protection is no protection. On the other hand, when $\underline{z} = 0$, that is, when $c \leq \alpha$, the optimal protection is no protection if and only if $SW(0) - SW(z^M) = (1-\alpha)\{(2\alpha + 6\alpha^2)c + (3 + 5\alpha)c^2 - \alpha^2(1-\alpha)\}/8\alpha(1+\alpha)^2 \geq 0$. Therefore, we have the following proposition for the underutilization problem.

Proposition 5. *The optimal level of copyright protection for the underutilization problem is full protection if*

$$(2\alpha + 6\alpha^2)c + (3 + 5\alpha)c^2 < \alpha^2(1-\alpha)$$

holds. Otherwise, it is no protection.

Fig. 3 shows the optimal protection for possible values of the parameters. As can be seen from the figure, for many values of the parameters, we confirm the general perception that the increased copyright protection will increase the social welfare loss due to underutilization by strengthening the monopoly power of the original producer. (When the marginal cost is greater than 0.048, it is always optimal to have no protection. We want to note, however, that the absolute magnitude is not very significant. This is so because we have normalized the parameters such that both v and c are less than 1.) On the other hand, we also find out that the increased copyright protection will decrease the social welfare loss due to underutilization when the marginal cost of the original producer is low enough relative to the parameter α, which measures the non-substitutability of the copy.

The reason for this is as follows: the increased copyright protection will affect

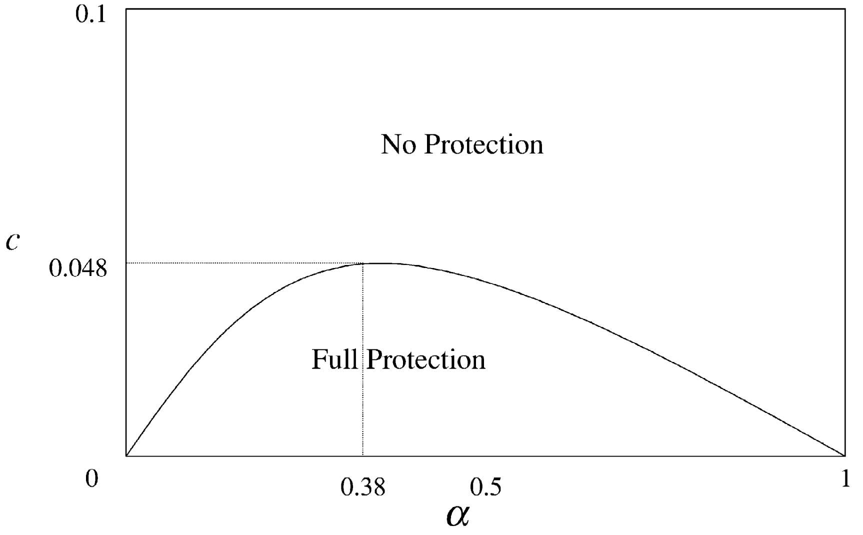

Fig. 3. The optimal level of copyright protection for the underutilization problem.

consumers' choices in a way that some consumers switch from making reproductions to buying from the original producer. Since it is more efficient that the product be provided by the original producer rather than by reproduction when the marginal cost c is low enough, this will lead to an increase in the social welfare.[9]

More generally, since the social welfare is a convex function which achieves an interior minimum at $z=(1-\alpha)(\alpha+3c)/(3+\alpha)$, the social welfare loss due to underutilization will decrease at the levels of copyright protection higher than $(1-\alpha)(\alpha+3c)/(3+\alpha)$. This implies that, even when the development cost is non-existent such that we need not protect the producer from financial loss, it is sometimes desirable to impose strong copyright protection. In other words, the copyright system may be used as an instrument to choose the efficient modes of production.[10]

3.2. The first stage: production

We now deal with the producer's decision to develop the copyrightable product in the first stage. The producer will develop the product if and only if the profit π in the second stage covers the development cost D. Since the profit is an increasing function of z, we conclude that the producer develops the product as long as the the level of copyright protection exceeds a threshold level denoted by z^*. We study how z^* is determined for different levels of D below. Firstly, if the development cost D is less than $\pi(\underline{z})$, where recall that $\underline{z}=\max\{0,\ c-\alpha\}$ represents the minimum level of copyright protection, then $z^*=\underline{z}$. Secondly, if the development cost lies between $\pi(\underline{z})$ and the maximum possible level of profit $\pi((1-\alpha)(1+c)/2)=(1-c)^2/4$, then we can get z^* by solving $\pi(z^*)=D$. Since π is a strictly increasing function of z, it follows that z^* is a strictly increasing function of D.[11] Lastly, if the development cost D is greater than $(1-c)^2/4$, then no level of copyright protection can induce development.

We are ready to discuss the optimal level of copyright protection, that is, the level of copyright protection which maximizes the social welfare given consumers' and the producer's optimal choices. Observe that the optimal level of copyright protection can be succinctly expressed as

$$\operatorname{argmax}_z SW(z) \text{ subject to } \pi(z) \geq D.$$

[9] It is appropriate to allude to one of Novos and Waldman's (1984) results here. They have a proposition (Proposition 3) which demonstrates: for any level of copyright protection, there exists a higher level of copyright protection such that any protection level higher than the latter will induce a smaller social welfare loss due to underutilization. Since they assume that consumers' reproduction costs are always higher than the original producer's marginal cost, their result is consistent with ours. Besen (1986) also obtains a similar result for the indirect appropriability case.

[10] We note, however, that the current analysis ignores the costs related to the maintenance of copyright system, such as enforcement costs. See the next section for more discussion.

[11] When $\pi(\underline{z}) \equiv (\alpha+\underline{z}-c)^2/4\alpha \leq D < \alpha(1-c)^2/(1+\alpha)^2 \equiv \pi(z^M)$, we have $z^*=(c-\alpha)+2\sqrt{\alpha D}$. When $\alpha(1-c)^2/(1+\alpha)^2 \leq D \leq (1-c)^2/4$, we have $z^*=(1-\alpha)(1+c-\sqrt{(1+c)^2-4(c+D)})/2$.

1. When $D > (1-c)^2/4$: in this case, the product will not be developed under any level of copyright protection.
2. When $\pi(z^M) < D \le (1-c)^2/4$: in this case, the optimal level of copyright protection is $\pi^{-1}(D)$, that is, the level z at which $\pi(z) = D$. Note that, under this level, (i) the producer's overall profit net of development cost is zero, and (ii) there exists no unauthorized reproduction of the copyrightable product.
3. When $D \le \pi(z^M)$: we need to consider two subcases here.
   (3-1) When $SW(\underline{z}) < SW(z^M)$: the optimal level of copyright protection is z^M.
   (3-2) When $SW(\underline{z}) \ge SW(z^M)$: we first define $z^0 = (1-\alpha)[(3+5\alpha)c - \alpha]/(1+\alpha)(3+\alpha)$. z^0 is the level of reproduction cost other than z^M, which attains the interior maximum level of the social welfare, $(1+2\alpha)(1-c)^2/2(1+\alpha)^2$. (Refer to Fig. 2.) Since $SW(\underline{z}) \ge SW(z^M)$, we know that $\underline{z} \le z^0$. In this case, if $D > \pi(z^0)$, then the optimal level of copyright protection is z^M. Otherwise, the optimal level of copyright protection is $z^D = \pi^{-1}(D)$ when $D > \pi(\underline{z})$ and $\underline{z}$ when $D \le \pi(\underline{z})$. Note that z^D necessarily lies between $\underline{z}$ and z^0.

Summarizing the discussion, we have the following important proposition.

Proposition 6. *If $SW(\underline{z}) \ge SW(z^M)$ and $D \le \pi(\underline{z})$, then the optimal level of copyright protection is no protection; if $SW(\underline{z}) \ge SW(z^M)$ and $\pi(\underline{z}) < D \le \pi(z^0)$, then it is the level at which the producer's second-stage profit exactly covers the first-stage development cost. In all other cases, the optimal level of copyright protection is full protection, that is, the level under which no unauthorized reproduction exists.*

Proposition 6 says that the optimal level of copyright protection is: (i) no protection, (ii) very low protection (z^D) under which the producer's overall profit is zero, or (iii) full protection under which no unauthorized reproduction of the copyrightable product exists. Therefore, no intermediate level of copyright protection is an optimal level. This *all-or-nothing* nature of the optimal level of copyright protection is clearly due to the behavior of the social welfare.

Proposition 6 provides all the relevant information for the decision of the optimal level of copyright protection for different values of parameters α, c, and D. We sketch some important characteristics below. We first observe that the optimal protection is full protection for parameter values α and c (regardless of D) at which full protection is the optimal protection for the underutilization problem. (Refer to Fig. 3.) In particular, for products such as software whose marginal cost c is essentially zero, the optimal level of copyright protection is full protection. We also observe that no protection can hardly be an optimal level of protection: when $\underline{z} > 0$, that is, when $c > \alpha$, we have that $\pi(\underline{z}) = 0$. Thus, for any positive development cost, no protection cannot be an optimal protection. When $\underline{z} = 0$, that is, when $c \leq \alpha$, no protection cannot be the optimal protection for $D > \pi(\underline{z}) \equiv (\alpha - c)^2/4\alpha$.

The social welfare loss due to underproduction, that is, the welfare loss that occurs when the producer does not develop the product due to insufficient copyright protection even though the social welfare from the development is positive, is decreasing as the level of copyright protection increases. The change, however, is not gradual. For a given development cost, the social welfare loss due to underproduction exists and remains the same as long as the level of copyright protection is less than z^* at which the second-stage profit is just equal to the first-stage development cost; and the social welfare loss becomes zero at the protection levels greater than z^*. Therefore, the social welfare loss due to underproduction as a function of the level of copyright protection is a step function, with a downward jump at z^* which is determined by the development cost.[12]

4. Discussion

We can extend the model to see how the optimal level of copyright protection should be set under the many firms setting. Suppose there exist many firms each of which may produce a specific copyrightable product. Consumers have unit demand for every product. We assume that the products are neither substitutes nor complements. The reproduction costs and the substitutability of a copy for each consumer are the same across different products. The firms are different only in that they have different development costs. (The firms' marginal cost of production is the same across the products.) Let F be the distribution function of the development cost D. Then, for a given level of copyright protection z, firms with $D \leq \pi(z)$ will develop their respective products, and firms with $D > \pi(z)$ will not develop the products. The aggregate social welfare, denoted by $SW^A(z)$, is then proportional to $SW(z) \cdot F(\pi(z))$. It is obvious that, when $SW(\underline{z}) < SW(z^M)$ holds, the optimal level of copyright protection for the society is full protection. On the other hand, when $SW(\underline{z}) > SW(z^M)$ holds, the optimal level depends on the shapes of the social welfare function and the profit function as well as on the distribution of the development cost. We first observe: if the social welfare is increasing at a particular level z, then z cannot be an optimal level since both the distribution function F and the profit function are increasing functions of z. This implies that the levels of copyright protection not less than $(1 - \alpha)(\alpha + 3c)/(3 + \alpha)$, except for

[12] Of course, if the development cost exceeds the maximum possible profit level $(1 - c)^2/4$, then the function is constant since no level of copyright protection can induce development. In addition, if we allow stochastic development costs, then the social welfare loss due to underproduction may become a smooth function, not a step function.

full protection, cannot be an optimal level.[13] On the other hand, any level of copyright protection less than $(1-\alpha)(\alpha+3c)/(3+\alpha)$ can be an optimal level depending on the distribution of firms' development costs. One implication of this analysis is that, when we are constrained to choose an optimal level of copyright protection for the society as a whole, then there may exist firms with strictly positive profits (net of development costs). These are firms with relatively low development costs.

We end the discussion by pointing out some limitations of this paper and directions for further research. Firstly, the model in this paper does not incorporate the maintenance costs of the copyright system, such as the monitoring costs, the enforcement costs, the administrative costs, and so on. The reason is partly due to the difficulty of comparing two substantially different sources of costs, and partly due to our focus on the core aspects of copyright protection. If we incorporate the maintenance costs that are increasing with the protection level, then the optimal level of copyright protection needs not be full protection. Secondly, this paper does not consider the sharing schemes among consumers. That is, we do not allow consumers to form groups to share the product among themselves. If this is possible, then the producer can charge a higher price for each product since each consumer in the group will contribute to the purchase of the product.[14] The analysis of Varian (2000) and Bakos et al. (1999) on consumer sharing may cast some light on the study of the optimal level of copyright protection.[15] Another aspect that this paper does not consider is demand network externality. It is an interesting, though not straightforward, research agenda to determine the optimal level of copyright protection when consumer valuations for a product increase as the total number of consumers who use the same product grows. Again, the analysis of Conner and Rumelt (1991) and Takeyama (1994) may cast some light on this important issue. Lastly, the model in this paper only considers the individual reproduction by the final consumers. While we believe that this form of unauthorized reproduction gains more importance in the Internet Age (as exemplified by the Napster case), an analysis on the optimal level of copyright protection for the alternative case of wholesale piracy where fringe firms sell the unauthorized copies to final consumers still deserves attention.

Acknowledgements

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[13] Recall that the social welfare achieves an interior minimum at $z=(1-\alpha)(\alpha+3c)/(3+\alpha)$.

[14] This is known as indirect appropriability since the producer can indirectly collect rewards from the consumers who do not buy the product directly from him. Libraries and video rental stores are examples of consumer sharing.

[15] See also the earlier works of Liebowitz (1985) and Besen and Kirby (1989) among others.

referees for many helpful comments. I am especially indebted to Chongmin Kim for invaluable suggestions.

Appendix A. A generalization

We show the robustness of the basic model in the text by showing that the behavior of relevant variables in a generalized environment is basically the same as that in the basic model. We consider a generalization of the basic model by relaxing the assumption that the reproduction costs z_i's are identical across consumers ($z_i(y) = z(y)$ for all i and y). Specifically, we assume that (i) the mass of consumers is 1, and (ii) the pair (v, z) of valuation and the reproduction cost is uniformly distributed over the rectangle $[0, 1] \times [z, z + m]$. That is, consumers' valuations are uniformly distributed over the unit interval [0, 1], the reproduction costs are uniformly distributed over the interval $[z, z + m]$, and they are independent. The parameter m represents the variation of the reproduction costs. As before, we assume that $\partial z / \partial y > 0$.

From the definition of the set $B(p; y) = \{i \in I \mid p \leq v_i, p \leq w_i\}$ of consumers who buy the product directly from the producer, we know that the mass of $B(p; y)$ is given by

$$b(p; y) = \int_{p}^{1} \int_{\max\{p, z+\alpha v\}}^{\max\{p, z+\alpha v+m\}} \frac{1}{m} \, \mathrm{d}w \, \mathrm{d}v.$$

Similarly, the mass of the set $N(p; y) = \{i \in I \mid v_i < p, v_i < w_i\}$ of consumers who neither buy nor make reproductions is given by

$$n(p; y) = \int_{0}^{\min\{p, 1\}} \int_{\max\{v, z+\alpha v\}}^{\max\{v, z+\alpha v+m\}} \frac{1}{m} \, \mathrm{d}w \, \mathrm{d}v.$$

The mass of the set $U(p; y) = \{i \in I \mid w_i \leq v_i, w_i < p\}$ of consumers who make reproductions is given by

$$u(p; y) = \begin{cases} \displaystyle\int_{z/(1-\alpha)}^{\min\{p, z+\alpha+m\}} \int_{\max\{w, (w-z-m)/\alpha\}}^{\min\{(w-z)/\alpha, 1\}} \frac{1}{m} \, \mathrm{d}v \, \mathrm{d}w & \text{for } p \geq z/(1-\alpha), \\ 0 & \text{for } p < z/(1-\alpha). \end{cases}$$

It is rather cumbersome to derive the exact quantities of the masses. Below we report only the results. The results are reported for the following three mutually exclusive cases: (1) when $\alpha + z + m/\alpha \leq 1$, (2) when $\alpha + z + m/\alpha > 1$ and $\alpha + z + m \leq 1$, and (3) when $\alpha + z + m > 1$.

[1] When $\alpha + z + m/\alpha \leq 1$:

$$
\begin{aligned}
b(p;y) = {} & (1-p)\cdot 1[p \leq z/(1-\alpha)] \\
& + \left(1 - \frac{z^2}{2\alpha m} - \left(1 + \frac{z}{m} - \frac{z}{\alpha m}\right)p - \frac{(1-\alpha)^2}{2\alpha m} - p^2\right) \\
& \cdot 1[z/(1-\alpha) < p \leq (z+m)/(1-\alpha)] \\
& + \left(1 + \frac{z}{\alpha} + \frac{m}{2\alpha} - \frac{p}{\alpha}\right)\cdot 1[(z+m)/(1-\alpha) < p \leq \alpha + z] \\
& + \frac{(\alpha + z + m - p)^2}{2\alpha m}\cdot 1[\alpha + z < p \leq \alpha + z + m],
\end{aligned}
$$

$$
\begin{aligned}
n(p;y) = {} & p\cdot 1[p \leq z/(1-\alpha)] \\
& + \left(-\frac{z^2}{2(1-\alpha)m} + \left(1 + \frac{z}{m}\right)p - \frac{1-\alpha}{2m}p^2\right) \\
& \cdot 1[z/(1-\alpha) < p \leq (z+m)/(1-\alpha)] \\
& + \frac{2z+m}{2(1-\alpha)}\cdot 1[(z+m)/(1-\alpha) < p].
\end{aligned}
$$

[2] When $\alpha + z + m/\alpha > 1$ and $\alpha + z + m \leq 1$:

$$
\begin{aligned}
b(p;y) = {} & (1-p)\cdot 1[p \leq z/(1-\alpha)] \\
& + \left(1 - \frac{z^2}{2\alpha m} - \left(1 + \frac{z}{m} - \frac{z}{\alpha}\right)p - \frac{(1-\alpha)^2}{2\alpha m}\, p^2\right) \\
& \cdot 1[z/(1-\alpha) < p \leq \alpha + z] \\
& + \left(1 + \frac{z}{m} + \frac{\alpha}{2m} - \left(1 + \frac{1}{m} + \frac{z}{m}\right)p + \left(\frac{1}{m} - \frac{\alpha}{2m}\right)p^2\right) \\
& \cdot 1[\alpha + z < p \leq (z+m)/(1-\alpha)] \\
& + \frac{(\alpha + z + m - p)^2}{2\alpha m}\cdot 1[(z+m)/(1-\alpha) < p \leq \alpha + z + m],
\end{aligned}
$$

$$
\begin{aligned}
n(p;y) = {} & p\cdot 1[p \leq z/(1-\alpha)] \\
& + \left(-\frac{z^2}{2(1-\alpha)m} + \left(1 + \frac{z}{m}\right)p - \frac{1-\alpha}{2m}\, p^2\right) \\
& \cdot 1[z/(1-\alpha) < p \leq (z+m)/(1-\alpha)] \\
& + \frac{2z+m}{2(1-\alpha)}\cdot 1[(z+m)/(1-\alpha) < p].
\end{aligned}
$$

[3] When $\alpha + z + m > 1$:

$$
\begin{aligned}
b(p;y) = {} & (1-p)\cdot 1[p \leq z/(1-\alpha)] \\
& + \left(1 - \frac{z^2}{2\alpha m} - \left(1 + \frac{z}{m} - \frac{z}{\alpha m}\right)p - \frac{(1-\alpha)^2}{2\alpha m}p^2\right) \\
& \cdot 1[z/(1-\alpha) < p \leq \alpha + z] \\
& + \left(1 + \frac{z}{m} + \frac{\alpha}{2m} - \left(1 + \frac{1}{m} + \frac{z}{m}\right)p + \left(\frac{1}{m} - \frac{\alpha}{2m}\right)p^2\right)\cdot 1[\alpha + z < p \leq 1],
\end{aligned}
$$

$$n(p;\,y) = p\cdot 1[p\le z/(1-\alpha)] + \left(-\frac{z^2}{2(1-\alpha)m} + \left(1+\frac{z}{m}\right)p - \frac{1-\alpha}{2m}\,p^2\right)\cdot 1[z/(1-\alpha) < p \le 1] + \left(-\frac{z^2}{2(1-\alpha)m} + 1 + \frac{z}{m} - \frac{1-\alpha}{2m}\right)\cdot 1[1<p].$$

In the above expression, $1[\cdot]$ is the indicator function. We can show that $b(p;\,y)$ and $n(p;\,y)$ as functions of p are continuous and have continuous first derivatives. Therefore, the function $u(p;\,y)$, which can be calculated as $u(p;\,y) = 1 - b(p;\,y) - n(p;\,y)$, has similar properties. In addition, $b(p;\,y)$ is decreasing and $n(p;\,y)$ is increasing.

It is impossible to analytically derive the producer's and consumers' optimal behaviors. We therefore rely on numerical analysis to see how the qualitative features of the basic model are affected in the generalized environment. We report below the simulation results for the typical case when $\alpha = 0.2$; $m = 0.5$; and $c = 0.1$. In the description below, z is the reproduction cost, which indirectly represents the level of copyright protection; p is the profit-maximizing price; $b(p;\,y)$ is the profit-maximizing quantity, which is nothing but the mass of consumers who buy from the producer; $u(p;\,y)$ is the mass of consumers who make reproductions; $n(p;\,y)$ is the mass of consumers who neither buy nor make reproductions; π is the maximized profit; CS is the consumer surplus; and SW is the social welfare, which is the sum of firm's profit and the consumer surplus (Table A.1).

The behavior of the social welfare in the generalized environment is similar to that in the basic model: the social welfare achieves local maxima at two points. The boundary maximum is achieved when $z = 0$, with a value 0.3542, and the interior maximum is achieved when $z = 0.175$, with a value 0.350. The behavior of the social welfare is shown in Fig. A.1.

The difference is that, while the interior maximum is achieved at the level of copyright protection under which unauthorized reproduction of the copyrightable product vanishes in the basic model, there still exist some reproduction activities in the generalized environment. For the given specification of the parameters ($\alpha = 0.2$; $m = 0.5$; $c = 0.1$), the interior maximum is achieved when $z = 0.175$ and the mass of consumers who make reproductions, that is, $u(p;\,y)$ is 0.097.

We also see from the table that the behavior of other variables exhibits the same pattern as that in the basic model: p is increasing, $b(p;\,y)$ is increasing at first and then decreasing, $u(p;\,y)$ is decreasing, $n(p;\,y)$ is increasing, π is increasing, and CS is decreasing. When $z \ge 0.44$, $u(p;\,y) = 0$ and the variables become stable.

As we can find from another numerical analysis for the case when $\alpha = 0.2$; $m = 0.5$; and $c = 0.0$, the shape of the social welfare critically depends on the relative magnitude of the producer's marginal cost as in the basic model: if the marginal cost is low enough, then the interior maximum is bigger. Otherwise, the boundary maximum is bigger (Table A.2).

Table A.1
Summary results ($\alpha = 0.2$; $m = 0.5$; $c = 0.1$)

z	p	$b(p; y)$	$u(p; y)$	$n(p; y)$	π	CS	SW
0.000	0.3260	0.4133	0.3457	0.2410	0.0934	0.2608	0.3542
0.025	0.3340	0.4321	0.3072	0.2607	0.1011	0.2492	0.3503
0.050	0.3415	0.4513	0.2695	0.2792	0.1090	0.2387	0.3477
0.075	0.3485	0.4708	0.2326	0.2966	0.1170	0.2292	0.3462
0.100	0.3555	0.4899	0.1971	0.3130	0.1252	0.2205	0.3456
0.125	0.3620	0.5094	0.1625	0.3281	0.1335	0.2127	0.3461
0.150	0.3680	0.5294	0.1287	0.3419	0.1419	0.2060	0.3478
0.175	0.3745	0.5479	0.0970	0.3551	0.1504	0.1997	0.3500
0.200	0.3895	0.5482	0.0778	0.3739	0.1587	0.1893	0.3480
0.225	0.4050	0.5460	0.0613	0.3927	0.1665	0.1790	0.3456
0.250	0.4205	0.5422	0.0467	0.4112	0.1738	0.1693	0.3430
0.275	0.4365	0.5360	0.0344	0.4296	0.1804	0.1596	0.3400
0.300	0.4525	0.5283	0.0240	0.4477	0.1862	0.1504	0.3366
0.325	0.4690	0.5184	0.0158	0.4659	0.1913	0.1412	0.3325
0.350	0.4860	0.5065	0.0094	0.4841	0.1955	0.1322	0.3277
0.375	0.5035	0.4926	0.0048	0.5025	0.1988	0.1233	0.3221
0.400	0.5210	0.4776	0.0018	0.5206	0.2011	0.1147	0.3158
0.425	0.5390	0.4608	0.0002	0.5390	0.2023	0.1063	0.3086
0.450	0.5500	0.4500	0.0000	0.5500	0.2025	0.1013	0.3038
0.475	0.5500	0.4500	0.0000	0.5500	0.2025	0.1013	0.3038

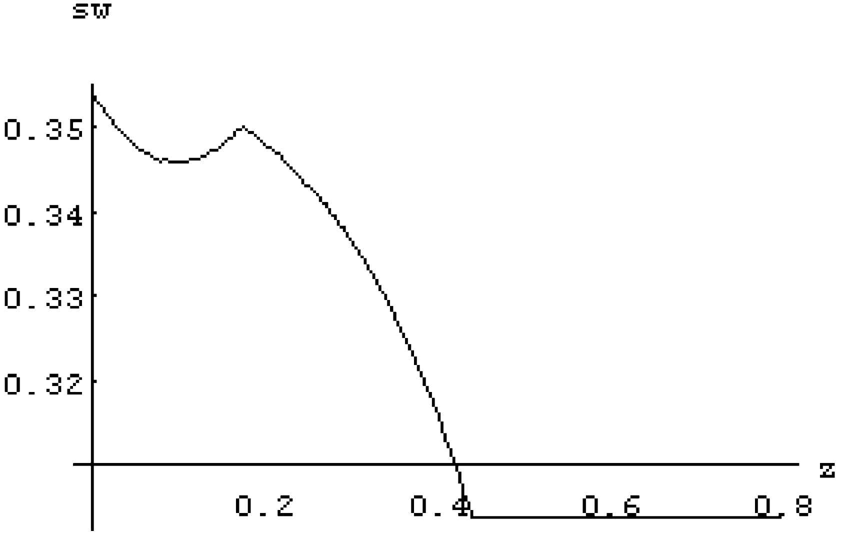

Fig. A.1. Social welfare ($\alpha = 0.2$; $m = 0.5$; $c = 0.1$).

Table A.2
Summary results ($\alpha = 0.2$; $m = 0.5$; $c = 0.0$)

z	p	$b(p; y)$	$u(p; y)$	$n(p; y)$	π	CS	SW
0.00	0.2615	0.5386	0.2546	0.2067	0.1408	0.2913	0.4322
0.02	0.2675	0.5556	0.2239	0.2205	0.1486	0.2833	0.4319
0.04	0.2730	0.5733	0.1935	0.2332	0.1565	0.2760	0.4325
0.06	0.2790	0.5896	0.1647	0.2457	0.1645	0.2690	0.4335
0.08	0.2840	0.6077	0.1353	0.2570	0.1726	0.2630	0.4356
0.10	0.2945	0.6136	0.1149	0.2715	0.1807	0.2541	0.4348
0.12	0.3070	0.6141	0.0986	0.2873	0.1885	0.2443	0.4328
0.14	0.3095	0.6137	0.0835	0.3028	0.1961	0.2348	0.4308
0.16	0.3320	0.6122	0.0697	0.3181	0.2033	0.2256	0.4288
0.18	0.3450	0.6089	0.0576	0.3335	0.2101	0.2164	0.4264
0.20	0.3580	0.6047	0.0467	0.3487	0.2165	0.2074	0.4239
0.22	0.3710	0.5995	0.0369	0.3636	0.2224	0.1988	0.4212
0.24	0.3845	0.5927	0.0286	0.3788	0.2279	0.1901	0.4179
0.26	0.3980	0.5849	0.0213	0.3937	0.2328	0.1816	0.4144
0.28	0.4120	0.5757	0.0154	0.4089	0.2372	0.1731	0.4103
0.30	0.4260	0.5657	0.0104	0.4239	0.2410	0.1649	0.4059
0.32	0.4405	0.5543	0.0066	0.4392	0.2441	0.1566	0.4007
0.34	0.4550	0.5421	0.0036	0.4543	0.2467	0.1485	0.3952
0.36	0.4695	0.5293	0.0015	0.4692	0.2485	0.1407	0.3892
0.38	0.4845	0.5152	0.0004	0.4844	0.2496	0.1329	0.3825
0.40	0.5000	0.5000	0.0000	0.5000	0.2500	0.1250	0.3750
0.42	0.5000	0.5000	0.0000	0.5000	0.2500	0.1250	0.3750

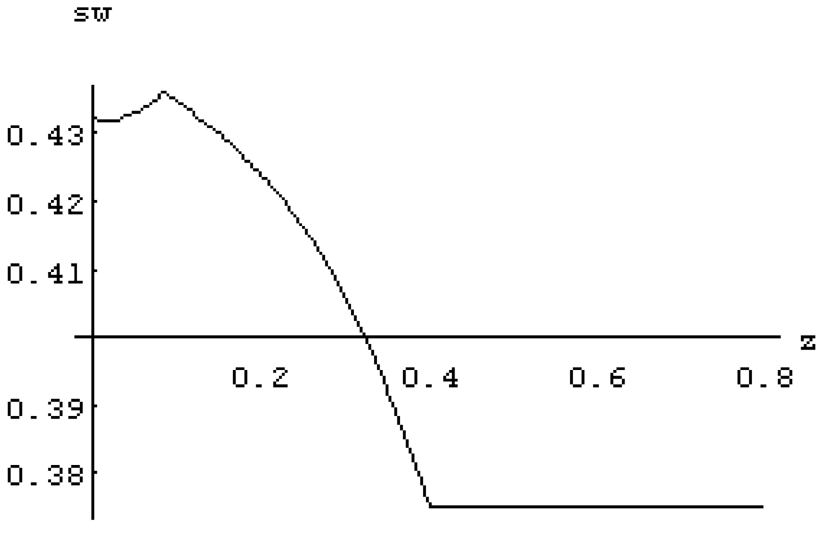

Fig. A.2. Social welfare ($\alpha = 0.2$; $m = 0.5$; $c = 0.0$).

The boundary maximum of the social welfare is attained at $z = 0.0$, with a value 0.4322. The interior maximum is attained at $z = 0.085$, with a value 0.436, where the mass $u(p;\ y)$ of consumers who make reproductions is 0.1285. This implies that, for the given specification, the interior maximum is achieved when the rate of unauthorized copying or the piracy rate, defined as $u(p;\ y)/(b(p;\ y) + u(p;y)) \times 100$, is $17.36\% = 0.1285/(0.6117 + 0.1285) \times 100$.

Observe that the variables become stable when $z \geq 0.4$. The behavior of the social welfare is shown in Fig. A.2.

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